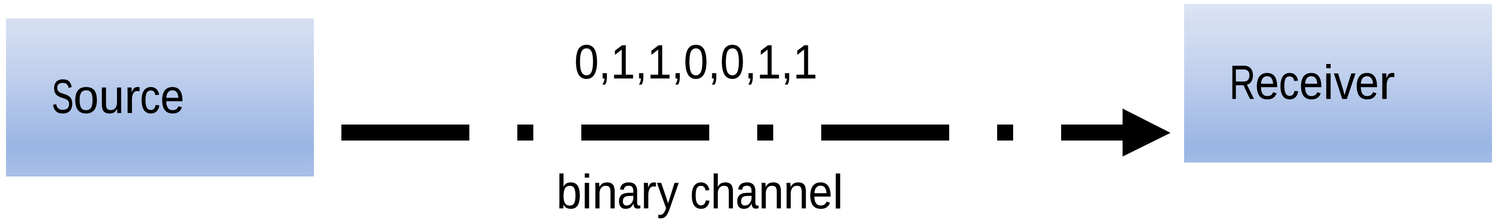
Channels

- A channel is used to get information across:

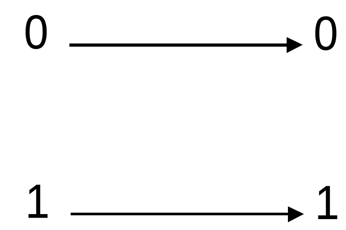


Many systems act like channels.
Some obvious ones: phone lines, Ethernet cables.
Less obvious ones: the air when speaking, TV screen when watching, paper when writing an article, etc.

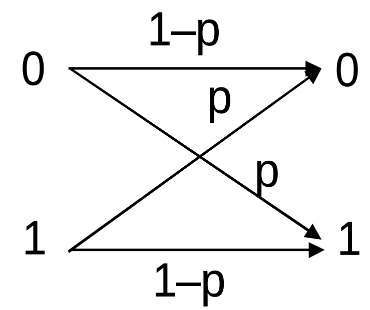
All these are physical devices and hence prone to errors.

Noisy Channels

A noiseless binary channel transmits bits without error:



A noisy, symmetric binary channel applies a bit-flip 0↔1 with probability p:



What to do if we have a noisy channel and you want to send information across reliably?

Error Correction pre-Shannon

- Primitive error correction (assume $p<1/2$):
  Instead of sending "0" and "1", send "0…0" and "1…1".
- The receiver takes the majority of the bit values as the 'intended' value of the sender.

- Example: If we repeat the bit value three times, the error goes down from p to $p^2(3-2p)$.
  Hence for $p=0.1$ we reduce the error to 0.028.
- However, now we have to send 3 bits to get one bit of information across and this will get worse if we want to reduce the error rate further…

Channel Rate

- When correcting errors, we have to be mindful of the rate of the bits that you use to encode one bit (in the previous example we had rate 1/3).

- For the primitive encoding in the previous example with $0 \rightarrow 0^r$ and $1 \rightarrow 1^r$ with rate $1/r$, the error goes down approximately as $p \rightarrow rp^{r-1}$.

- If we want to send data with arbitrarily small errors, then this requires arbitrarily low rates r, which is costly.

Error Correction by Shannon

- Shannon's basic observations:

- Correcting single bits is very wasteful and inefficient;
- Instead we should correct blocks of bits.

- We will see later that by doing so we can get arbitrarily small errors for the constant channel rate $1-H(p)$ where $H(p)$ is the Shannon entropy, defined by $H(p) = -p \log_2(p) - (1-p) \log_2(1-p)$.

A model for a Communication System

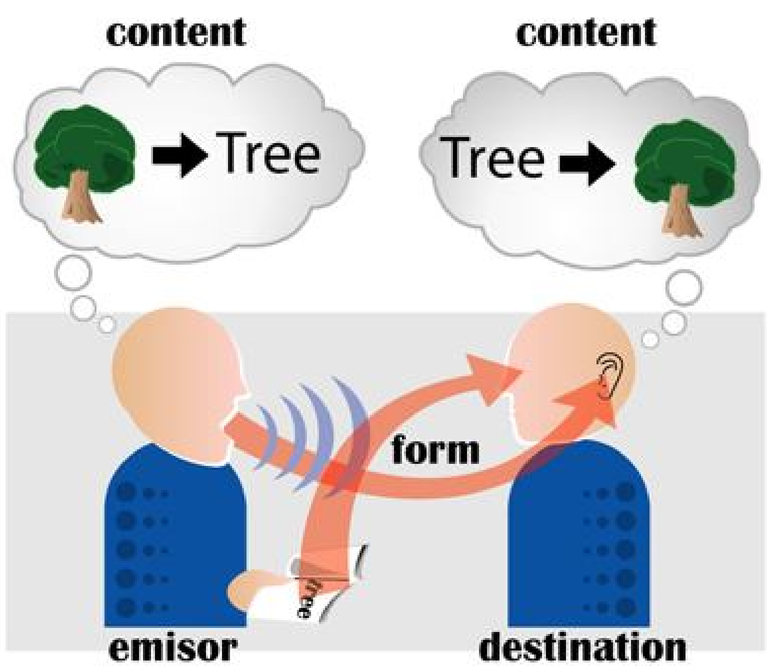
- The communications systems are of statistical nature.
- That is, the performance of the system can never be described in a deterministic sense; it is always given in statistical terms.
- A source is a device that selects and transmits sequences of symbols from a given alphabet.
- Each selection is made at random, although this selection may be based on some statistical rule.

A model for a Communication System

- The channel transmits the incoming symbols to the receiver. The performance of the channel is also based on laws of chance.
- If the source transmits a symbol A, with a probability of P{A} and the channel lets through the letter A with a probability denoted by P{A|A}, then the probability of transmitting A and receiving A is P{A}·P{A|A}

A model for a Communication System

- The channel is generally lossy: a part of the transmitted content does not reach its destination or it reaches the destination in a distorted form.



A model for a Communication System

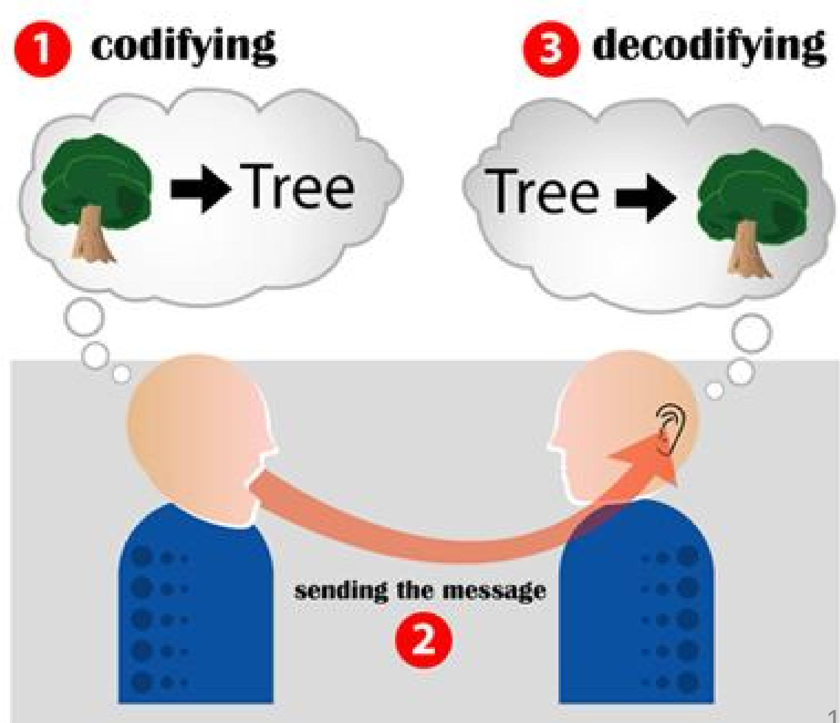
- A very important task is the minimization of the loss and the optimum recovery of the original content when it is corrupted by the effect of noise.
- A method that is used to improve the efficiency of the channel is called encoding.
- An encoded message is less sensitive to noise.

A model for a Communication System

- Decoding is employed to transform the encoded messages into the original form, which is acceptable to the receiver.

➢Encoding: $F: I \rightarrow F(I)$

➢Decoding: $F^{-1}: F(I) \rightarrow I$



A Quantitative Measure of Information

- Suppose we have to select some equipment from a catalog which indicates n distinct models: $\{x_1, x_2, ..., x_n\}$
- The desired amount of information $I(x_k)$ associated with the selection of a particular model x_k must be a function of the probability of choosing x_k :

$$I(x_k) = f\left(P\{x_k\}\right)$$

A Quantitative Measure of Information

- If, for simplicity, we assume that each one of these models is selected with an equal probability, then the desired amount of information is only a function of n:

$$I_1(x_k) = f(1/n)$$

A Quantitative Measure of Information

- If each piece of equipment can be ordered in one of m different colors and the selection of colors is also equiprobable, then the amount of information associated with the selection of a color c_j is :

$$I_2\left(c_j\right) = f\left(P\left\{c_j\right\}\right) = f\left(1/m\right)$$

A Quantitative Measure of Information

- The selection may be done in two ways:
- Select the equipment and then the color independently of each other

$$I\left(x_k \,\&\, c_j\right) = I_1(x_k) + I_1(c_j) = f\left(1/n\right) + f\left(1/m\right)$$

- Select the equipment and its color at the same time as one selection from *mn* possible choices:

$$I\left(x_k \,\&\, c_j\right) = f\left(1/mn\right)$$

A Quantitative Measure of Information

- Since these amounts of information are identical, we obtain:

$$f\left(1/n\right)+f\left(1/m\right)=f\left(1/mn\right)$$

- Among several solutions of this functional equation, the most important for us is:

$$f\left(x\right)=-\log\left(x\right)$$

- Thus, when a statistical experiment has n equiprobable outcomes, the average amount of information associated with an outcome is $\log n$

A Quantitative Measure of Information

- The logarithmic information measure has the desirable property of additivity for independent statistical experiments.
- The simplest case to consider is a selection between two equiprobable events. The amount of information associated with the selection of one out of two equiprobable events is $-\log_2 1/2 = \log_2 2 = 1$ and provides a unit of information known as a **bit**.